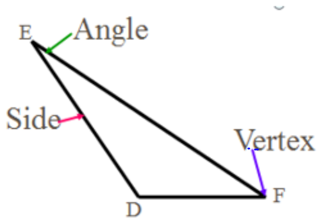
Angle Relationships in Triangles

A **triangle** is a figure formed when three noncollinear (not on the same line) points are connected by segments.



The sides are: $\overline{EF}, \overline{DE}, \overline{FD}$

The vertices are: E, F, D

The angles are: $\angle E, \angle F, \angle D$

Opposite Side of $\angle$F: $\overline{ED}$

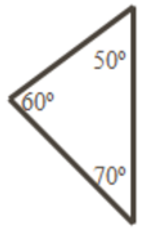
Opposite Side of $\angle$E: $\overline{DF}$

Opposite Side of $\angle$D: $\overline{EF}$

Triangles can be classified by two categories: **by Angles and by Sides.**

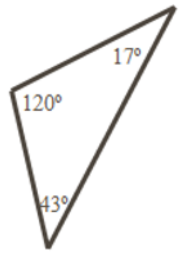
ANGLES

Acute



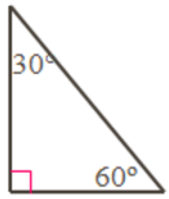
All Acute Angles

Obtuse



One Obtuse Angle

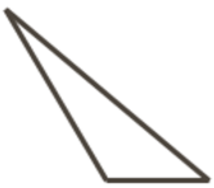
Right



One Right Angle

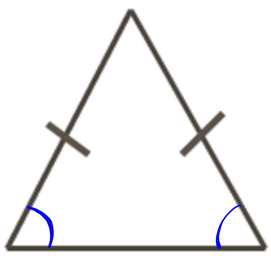
SIDES

Scalene



No Sides Congruent

Isosceles

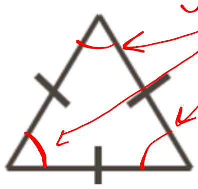


At Least 2 Sides Congruent

2 angles

Equilateral

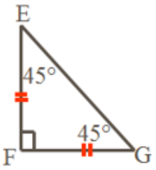
$\frac{180}{3} = 60°$



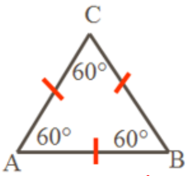
All Sides Congruent

All angles Too!

Practice: Classify the triangles by sides and angles.



- Right
- Isosceles



- Equilateral

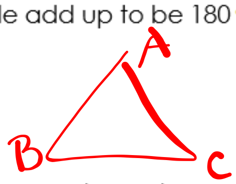
Think About It: Check which triangles are possible.

	Acute	Obtuse	Right
Scalene			
Isosceles			
Equilateral			

Triangle Sum Theorem

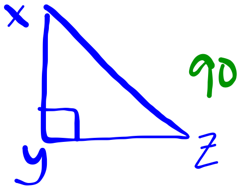
<u>Triangle Sum Theorem</u>: The measures of the three interior angles in a triangle add up to be 180°

This means: $\angle A + \angle B + \angle C = 180$



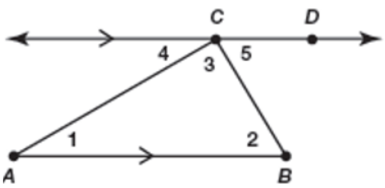
<u>Corollary to Triangle Sum Theorem</u>: The acute angles of a right triangle are complementary.

This means: $\angle X + \angle Z = 90°$



$90 + 90 = 180$

Proof of the Triangle Sum Theorem:

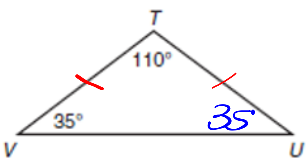


Given: Triangle *ABC* with $\overline{AB} \parallel \overline{CD}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

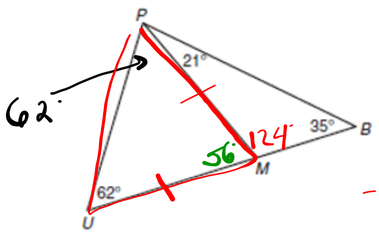
Statements	Reasons
1. ______	1. Given
2. $\angle 4 \cong \angle 1$	2. ______
3. ______	3. Alt. Interior Angles are $\cong$
4. $m\angle 4 \cong m\angle 1$	4. ______
5. ______	5. Def. of $\cong$ Angles
6. $m\angle ACD = m\angle 5 + m\angle 3$	6. ______
7. $m\angle 4 + m\angle ACD = 180°$	7. Linear Pair Postulate
8. ______	8. Substitution Property
9. ______	9. Substitution Property

Examples: Find m∠U.



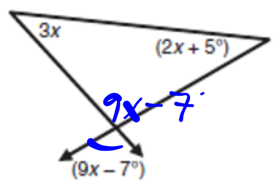
$180 - 110 - 35 = 35$

Find m∠UPM



$180 - 21 - 35 = 124$

Find the measure of x.



$3x + 2x + 5 + 9x - 7 = 180$

$14x - 2 = 180$

$14x = 182$

$x = 13$

Exterior Angle Theorem

Exterior Angle Theorem: The measure of the exterior angle is equal to the sum of two remote interior angles.

Interpret: What does exterior mean? outside

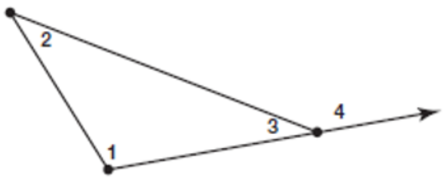
What does interior mean? inside

What does remote mean? away from

Ext. = remote + remote

Proof: Prove the **Exterior Angle Theorem**:

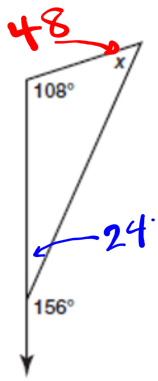
Given: ∠1, ∠2, and ∠3 are interior angles.
Prove: ∠4 = ∠1 + ∠2



Statements	Reasons
1. ∠1, ∠2, and ∠3 are int. angles.	1. ________
2. ________	2. Triangle Sum Theorem
3. ________	3. Definition of a Linear Pair
4. ________	4. Transitive/Substitution
5. ________	5. Subtraction Property

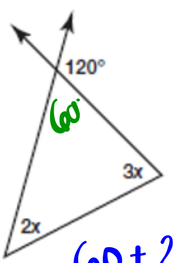
Examples:

A.



$180 - 108 - 24 = 48$

B.

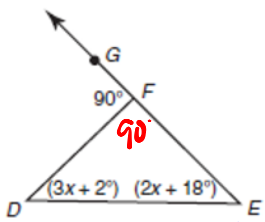


$60 + 2x + 3x = 180$

$5x = 120$

$x = 24$

C.



$3x + 2 + 2x + 18 + 90 = 180$

$5x + 110 = 180$

$5x = 70$

$x = 14$

Solve for x using the Exterior Angle Theorem:

a.

$156 = 108 + x$

$x = 48°$

b.

$120 = 2x + 3x$

$120 = 5x$

$x = 24$

c.

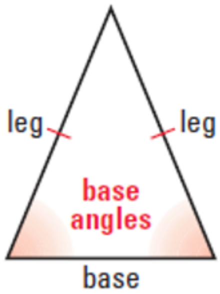
$90 = 3x + 2 + 2x + 18$

$90 = 5x + 20$

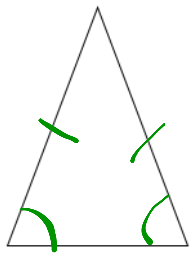
$5x = 70$

$x = 14$

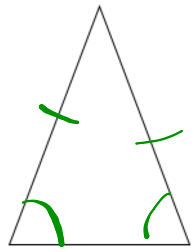
Isosceles Base Angle Theorem and Its Converse



Isosceles Triangle



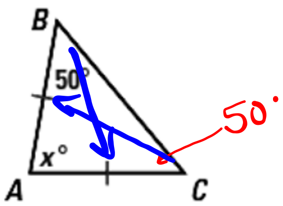
Base Angles Theorem:
If two sides of a triangle are congruent, then the angles opposite them are congruent.



Converse of Base Angles Theorem:
If two angles of a triangle are congruent, then the sides opposite of them are congruent.

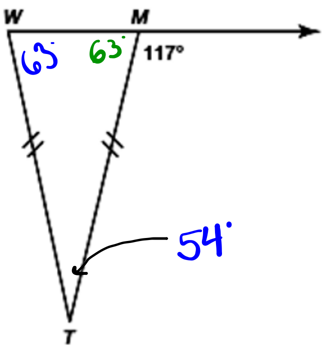
Examples:

A. Find the value of x

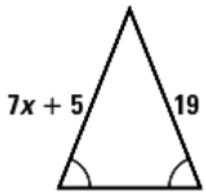


$x = 80°$

B. Find the m∠T

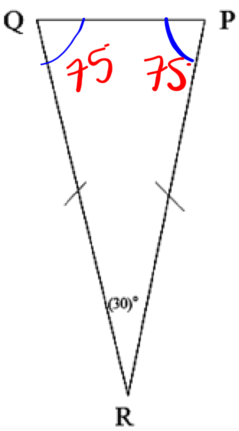


C. Find the value of x.



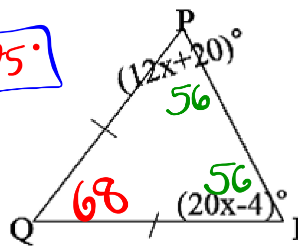
$7x + 5 = 19$
$7x = 14$
$x = 2$

D. Find the measure of <P.
∠R = 30°



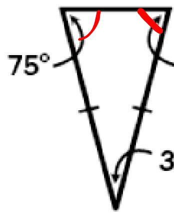
$\frac{150}{2} = \boxed{75°}$

E. Find the measure of ∠Q



$12x + 20 = 20x - 4$
$24 = 8x$
$\boxed{x = 3}$
$\boxed{\angle Q = 68°}$

F. Find the value of x & y.



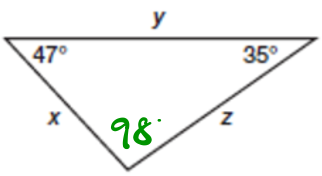
$5x° = 75$
$x = 15$
$3y° = 30$
$y = 10$

Side Inequality Theorem

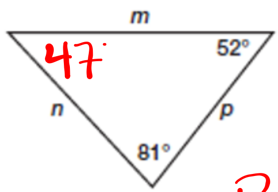
<u>Side Inequality Theorem:</u> If one side of a triangle is longer than the other side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

This means: The largest angle of a triangle lies opposite the longest side. The smallest angle lies opposite the shortest side. If two angles are equal, their side lengths will be equal.

Example: List the sides from shortest to longest for each diagram.



x, z, y



p, n, m