



Keeper #5: Solving Equations w/ Matrices

$$A = \begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}$$

0 - 2

$$B = \begin{bmatrix} 1 & -1 & 2 \\ 5 & 1 & -2 \\ 0 & -3 & -4 \end{bmatrix}$$

-4 + 0 - 30
0 + 6 + 20

1. |B|

- 8 -34
- 60 26

2. |A| -2

3. A^{-1}

$$\frac{1}{-2} \begin{bmatrix} 0 & -2 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 0 & +1 \\ +.5 & -2.5 \end{bmatrix}$$

4. B^{-1}

Augmented Matrices

A system of equations written in matrix form using coefficients of variables.

$$\begin{bmatrix} A \\ \text{coeff.} \end{bmatrix} \mid \text{answers} \quad \begin{bmatrix} A \\ \text{coeff.} \end{bmatrix} \begin{bmatrix} \text{variables} \end{bmatrix} = \begin{bmatrix} \text{answers} \end{bmatrix}$$

1. Write the coefficients of the x -terms as the numbers down the first column.
2. Write the coefficients of the y -terms as the numbers down the second column.
3. If there are z -terms, write the coefficients as the numbers down the third column.
4. Draw a vertical line and write the constants to the right of the line.

Ex. 1 set up into augmented matrix

$$3x - 2y = 14$$

$$x + 3y = 1$$

$$\begin{array}{cc} x & y \\ \left[\begin{array}{cc|c} 3 & -2 & 14 \\ 1 & 3 & 1 \end{array} \right] \end{array}$$

How to solve:

$$\cancel{A^{-1}} \cancel{A} \cdot X = B$$

$$X = A^{-1} \cdot B$$

$$\cancel{\begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 1 \end{bmatrix}$$

$$|A| = 11 \quad A^{-1} = \frac{1}{11} \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\frac{1}{11} \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 14 \\ 1 \end{bmatrix}$$

Ex. 2 solve for x and y

~~$y - 2x = -3$~~

$x - 4y = -2$

$-2x + y = -3$

$$\begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix} \cdot \frac{1}{-7} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

A^{-1}

$$\frac{1}{-7} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} -14 \\ -7 \end{bmatrix}$$

$$\begin{aligned} x &= 2 \\ y &= 1 \end{aligned}$$

$$\begin{aligned} x &= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \\ y &= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \end{aligned}$$

$$|A| = 1 - 8 = -7$$

$$A^{-1} = \frac{1}{-7} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

Ex. 3 solve for x and y

$3x - 6y = -9$

$-2y - 2x = 12$

Ex. 4 solve for x, y, and z

$3x + y - 2z = 2$

$x - 2y + z = 3$

$2x - y - 3z = 3$

Ex. 3 solve for x and y

$$3x - 6y = -9$$

~~$$-2y - 2x = 12$$~~

$$-2x - 2y = 12$$

$$\boxed{\begin{matrix} x = -5 \\ y = -1 \end{matrix}}$$

$$\begin{bmatrix} 3 & -6 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ 12 \end{bmatrix}$$

$$|A| = -18 \quad A^{-1} = \frac{1}{-18} \begin{bmatrix} -2 & 6 \\ 2 & 3 \end{bmatrix}$$

18 + 72

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-18} \begin{bmatrix} 90 \\ 18 \end{bmatrix}$$

$$1) \begin{bmatrix} 4 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$|A| = 13$$

$$A^{-1} = \frac{1}{13} \begin{bmatrix} 4 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -10 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 26 \\ -52 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$